

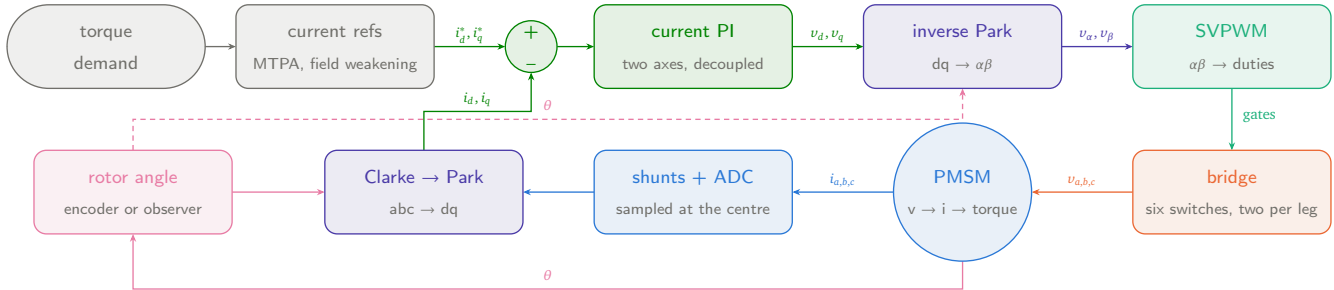
Field-oriented control

formula sheet

Amplitude-invariant scaling, per-phase parameters, d along the rotor flux. Worked numbers are computed from the same model the site's figures run on. Every formula name is a link to the page that derives it — start at <https://focreference.com>.

The loop

Every page on the site is somewhere on this diagram. It is drawn from the same specification the site draws from, so the two cannot disagree.



Frames

Clarke, amplitude-invariant

$$i_\alpha = \frac{2}{3} \left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right), \quad i_\beta = \frac{1}{\sqrt{3}} (i_b - i_c)$$

The 2/3 keeps the peak. With a floating neutral the third current is redundant.

Park

$$i_d = i_\alpha \cos \theta_e + i_\beta \sin \theta_e, \quad i_q = -i_\alpha \sin \theta_e + i_\beta \cos \theta_e$$

A rotation, so it preserves magnitude and the dot product — and therefore power.

Power

$$p = \frac{3}{2} (v_d i_d + v_q i_q)$$

The 3/2 is the amplitude-invariant convention, not a fudge.

The machine

dq voltage equations

$$v_d = R_s i_d + L_d \frac{di_d}{dt} - \omega_e L_q i_q$$

$$v_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_e (L_d i_d + \lambda_m)$$

The cross terms are what decoupling cancels; the λ_m term is the back-EMF.

Torque

$$T_e = \frac{3}{2} p [\lambda_m i_q + (L_d - L_q) i_d i_q]$$

Magnet term plus reluctance term. On this machine $L_q > L_d$, so negative i_d adds torque. Torque constant 0.051 N·m/A at $i_d = 0$.

► saliency $L_q/L_d = 1.57$

Characteristic current

$$i_{ch} = -\frac{\lambda_m}{L_d}$$

Decides the whole field-weakening envelope. A machine whose rated current reaches it can be weakened indefinitely.

► 70.8 A on the reference machine

Back-EMF constant

$$K_e = \frac{\sqrt{3}}{\sqrt{2}} \lambda_m \omega_{e,1000}$$

Line-to-line RMS per 1000 rpm — the form a datasheet quotes. Three conversions stand between it and λ_m .

The inverter

Linear voltage limit

$$|v|_{max} = \frac{V_{dc}}{\sqrt{3}}$$

The inscribed circle of the hexagon. SPWM reaches only $V_{dc}/2$ — 15.5% less.

► 27.7 V against 24 V at 48 V

SVPWM dwell times

$$t_1 = \frac{\sqrt{3} |v|}{V_{dc}} \sin\left(\frac{\pi}{3} - \gamma\right), \quad t_2 = \frac{\sqrt{3} |v|}{V_{dc}} \sin \gamma$$

$$t_0 = 1 - t_1 - t_2$$

γ is the angle within the sector. The switching order is decided by the switch words, not by which vector is trailing.

Deadtime voltage error

$$\Delta v = \frac{t_d}{T_s} V_{dc} \operatorname{sgn}(i)$$

Sign follows the current, so it is a square wave at the electrical frequency — 6× ripple in dq.

Bridge loss

$$P = 3 I_{rms}^2 R_{ds(on)} + 3 f_{pwm} V_{dc} |\overline{i}| t_{sw}$$

Square-law loss takes the RMS; linear loss takes the mean of the absolute sine, 0.6366. A lower $R_{ds(on)}$ moves the crossover down.

Control

Current-loop gains

$$k_p = \omega_c L, \quad k_i = \omega_c R_s$$

Pole-zero cancellation leaves a pure integrator, so the closed loop is first order at ω_c .

► $k_p = 1.319$, $k_i = 2199$ for 1 kHz on the d axis

Anti-windup tracking gain

$$k_{aw} = \frac{k_i}{k_p} = \frac{R_s}{L}$$

Back-calculation. Too low and the integrator still runs away, just slower.

Phase margin from delay

$$\text{PM} = 90^\circ - \omega_c T_d$$

The whole stability story of a well-tuned current loop. A timing property, not a gain property.

► 75 μs of delay gives 63° at 1 kHz

Angle prediction

$$\theta_{out} = \theta_{measured} + \omega_e T_d$$

Removes a speed-dependent rotation entirely. Does nothing for phase margin.

Cascade separation

$$\frac{\omega_{inner}}{\omega_{outer}} \geq 5 \text{ to } 10$$

10× costs nothing, 5× starts to cost, under 3× doubles overshoot.

Sensing

Low-side measurement window

$$t_{window} = (1 - d) T_s$$

Closes as duty approaches 1. The phase with the largest duty fails first.

Converter resolution

$$q = \frac{2I_{fs}}{2^N}, \quad \sigma = \frac{q}{\sqrt{12}}$$

In dq the RMS is $\sqrt{2/3}$ of the per-phase figure, and the peak bound is 2/3 of a code — not a half.

Encoder speed step

$$\Delta\omega = \frac{2\pi}{\text{CPR} \cdot T}$$

Independent of speed, so it grows as a fraction of the measurement as the machine slows.

Link capacitor current

$$I_{cap} = \sqrt{I_{rms}^2 - I_{mean}^2}$$

Exact, not approximate — mean and ripple are orthogonal. Peaks near half modulation depth.

The reference machine

Every worked number on this sheet is computed against these, from `src/core` at build time.

R_s	0.35 Ω	stator resistance, per phase
L_d	210 μH	direct-axis inductance
L_q	330 μH	quadrature-axis inductance
λ_m	8.5 mWb	rotor flux linkage, peak
p	4	pole pairs
V_{dc}	48 V	DC link, for the worked voltages

Numbers worth recognising

Read from the same named constants the figures use, so none of them can drift into being merely plausible.

0.6667	2/3	Clarke scale that keeps the peak
1.5000	3/2	resultant MMF, and the power and torque factor
1.1547	$2/\sqrt{3}$	what SVPWM is worth over SPWM
0.8660	$\sqrt{3}/2$	modulation index where plain SPWM starts to clip
0.5774	$1/\sqrt{3}$	linear voltage limit, as a fraction of V_{dc}
0.6366	$2/\pi$	mean of the absolute sine — every loss term linear in current
0.9549	$3/\pi$	six-step mean torque, against FOC at the same current
0.1403		six-step torque ripple, peak to peak over mean
0.8165	$\sqrt{2/3}$	per-phase quantisation noise carried into dq, RMS
0.6667	2/3	peak dq rounding error, in codes — not a half
0.00393		copper resistance, per kelvin — R_s rises 39% over 100 K
-0.00120		NdFeB flux, per kelvin — the magnet weakens as it heats